Analysis:

Elemat

Umformen:

1) $3a + (2b - 2a) = 3a + 2b - 2a = a + 2b$

2) $a - (b - (a + b - (c - 2a + b) + c)) = a - (b - (a + b - c + 2a - b + c))$
$= a - (b - a - b + c - 2a + b - c) = a - b + a + b - c + 2a - b + c$
$= 4a - b$

3) $(x-3)(x+2)(x-1) = (x^2 + 2x - 3x - 6)(x-1)$
$= x^3 + 2x^2 - 3x^2 - 6x - x^2 - 2x + 3x + 6 = x^3 - 2x^2 - 5x + 6$

4) $(2u - 2v + 3w)(u + 4v - 6w) = 2u^2 + 8uv - 12uw - 2uv$
$- 8v^2 + 12vw + 3uw + 12vw - 18w^2$
$= 2u^2 - 8v^2 - 18w^2 + 6uv - 9uw + 24vw$

5) $\frac{b}{a} - \frac{a}{b} = \frac{b^2}{ab} - \frac{a^2}{ab} = \frac{b^2 - a^2}{ab}$

6) $\frac{u-v}{uv} - \frac{u-w}{uw} + \frac{v-w}{vw} = \frac{(uw - vw) - (uv - vw) + (uv - uw)}{uvw}$

$= \frac{\cancel{uw} - \cancel{vw} - \cancel{uv} + \cancel{vw} + \cancel{uv} - \cancel{uw}}{uvw} = \frac{0}{uvw} = 0$

7) $\frac{28ac}{9bd} : \frac{7ab}{12cd} = \frac{28ac}{9bd} \cdot \frac{12cd}{7ab} = \frac{336\cancel{a}c^2\cancel{d}}{63\cancel{a}b^2\cancel{d}}$

$= \frac{112c^2}{21b^2} = \frac{16c^2}{3b^2}$

9) $$\frac{\frac{x}{y}+1}{\frac{x}{y}-\frac{y}{x}} = \frac{\frac{x+y}{y}}{\frac{x^2-y^2}{xy}} = \frac{x+y}{y} \cdot \frac{xy}{x^2-y^2}$$

$$= \frac{\cancel{x+y}}{\cancel{y}} \cdot \frac{x\cancel{y}}{\cancel{(x+y)}(x-y)} = \frac{x}{x-y}$$

10) $$\frac{\frac{1}{a+b}-\frac{1}{a-b}}{1-\frac{a}{a-b}} = \frac{\frac{a-b-a-b}{(a+b)(a-b)}}{\frac{a-b-a}{a-b}} =$$

$$= \frac{-2b}{(a+b)(a-b)} \cdot \frac{a-b}{-b} = \cancel{\frac{-b}{a+b}} \quad \frac{2}{a+b}$$

8) $\frac{4u^2-16v^2}{3(u+v)} \cdot \frac{3u+6v}{2u-4v} = \frac{\cancel{(2u-4v)}(2u+4v)}{\cancel{3}(u+v)} \cdot \frac{\cancel{3}(u+2v)}{\cancel{2u-4v}}$

$\frac{(2u+4v)(u+2v)}{u+v} = \frac{2u^2+4vu+4vu+8v^2}{u+v}$

$= \frac{2u^2+8v^2+8vu}{u+v} = \frac{2(u^2+4v^2+4vu)}{u+v} = \frac{2(u+2v)^2}{u+v}$

11) $\frac{ab+ac}{bd+cd} = \frac{a(b+c)}{d(b+c)} = \frac{a}{d}$

12) $\frac{a}{a+b} + \frac{b}{a-b} + \frac{2ab}{a^2-b^2} = \frac{(a\cdot(a-b)) + b(a+b) + 2ab}{a^2-b^2}$

$= \frac{a^2-ab+ab+b^2+2ab}{a^2-b^2} = \frac{a^2+b^2+2ab}{a^2-b^2} = \frac{(a+b)^2}{(a+b)(a-b)}$

$= \frac{a+b}{a-b}$

Umformen – Potenzen

13) $(-a)^4 + (-2b)^3 - 2b^3 + (-3a)^4$

$= a^4 - 8b^3 - 2b^3 + 81a^4 = 82a^4 - 10b^3$

14) $18(a-1)^3 - 3(1-a)^3 - 16(a-1)^3 - 4(1-a)^3 - 3(a-1)^3$

$= 18a^3 - 18 - 3 + 3a^3 - 16a^3 + 16 - 4 + 4a^3 - 3a^3 + 3$

$= 6a^3 - 6 = 6(a^3-1) = 6(a-1)^3$

15) $(b-4)a^3 + (b-2)a^3 - (2b-3)a^3 - (2b-4)a^3 + ba^3$

$= a^3b - 4a^3 + a^3b - 2a^3 - 2a^3b + 3a^3 - 2a^3b + 4a^3 + a^3b$

$= -a^3b + a^3 = a^3(1-b)$

16) $\dfrac{a^3 b a b^{-4}}{a^2 b^{-2} a^{-3}} = \dfrac{a^4 b \cdot b^2 \cdot a^3}{a^2 \cdot b^4} = \dfrac{a^7 \cdot b^3}{a^2 \cdot b^4} = a^5 b^{-1}$

17) $\dfrac{21 \cdot a^3 b^2 x^{n+1}}{18 c^3 y^2 z^{n-3}} \cdot \dfrac{27 c^2 y^3 z^{n-2}}{35 a^2 b^3 x^{n+2}}$

$(n+1) - (n+2) = -1$

$(n-2) - (n-3) = 2n+1$

$n-2-n+3 = 1$

$= \dfrac{a \cdot y \cdot 9 \cdot z}{b \cdot c \cdot 10 \cdot x} = \dfrac{9ayz}{10bcx}$

18) $\dfrac{a^{x-1} \cdot b^{n+1} + a^x b^n + a^{x+1} b^{n-1}}{1} \cdot \dfrac{1}{a^{x-1} b^{n-2}}$

$= \dfrac{a^x b^n (a^{-1} b + 1 + a b^{-1})}{a^{x-1} \cdot b^{n-2}} = a b^2 (a^{-1} b + 1 + a b^{-1})$

$b^3 + ab^2 + a^2 b = b(b^2 + ab + a^2) = b \cdot ((a+b)^2)$

19) $\dfrac{(ax+ay)^{n+1}\cdot b^{n}}{(abx+aby)^{n-1}}$

$\dfrac{(ax+ay)^{n+1}\cdot b^{n+0}}{(ax+ay)^{n-1}\cdot b^{n-1}} = (ax+ay)^2\cdot b$

$= a^2\cdot(x+y)^2\cdot b$

20) $3\cdot\sqrt[4]{256} - 4\sqrt{49} - 7\sqrt[3]{27} + 2\sqrt[5]{32}$

$= 3\cdot 256^{1/4} - 4\cdot 49^{1/2} - 7\cdot 27^{1/3} + 2\cdot 32^{1/5}$

$= 12 - 28 - 21 + 4 = -33$

21) $(2\sqrt{5a} - 5\sqrt{2b})(2\sqrt{5a} + 5\sqrt{2b})$

$= 4\sqrt{25a^2} + \cancel{10\sqrt{10ab}} - (\cancel{10\sqrt{10ab}}) - (25\sqrt{4b^2})$

$= 4\sqrt{25a^2} - (25\sqrt{4b^2} = 20a - 50b$

22) $\sqrt{a-\sqrt{a^2-b^2}} \cdot \sqrt{a+\sqrt{a^2-b^2}}$

$= \sqrt{(a-\sqrt{a^2-b^2})\cdot(a+\sqrt{a^2-b^2})} \longrightarrow$ Binomische Formel

$= \sqrt{a^2-(\sqrt{a^2-b^2})^2} = \sqrt{a^2-(a^2-b^2)} = \sqrt{a^2-a^2+b^2}$

$= \sqrt{b^2} = b$

23) $\sqrt[2n+1]{a^{4n^2-1}} = \left(\sqrt[2n+1]{a}\right)^{4n^2-1}$

$= \left(a^{\frac{1}{2n+1}}\right)^{4n^2-1} = a^{\frac{4n^2-1}{2n+1}}$

$= \frac{4n^2-1}{2n+1} = \frac{(2n-1)\cancel{(2n+1)}}{\cancel{(2n+1)}}$

$= 2n-1 \longrightarrow a^{2n-1}$

24) $\dfrac{\sqrt[3]{3\sqrt{3}}}{\sqrt[6]{3}} = \dfrac{3^{1/3}\cdot 3^{1/6}}{3^{1/6}}$

$= \dfrac{3^{1/3+1/6}}{3^{1/6}} = \dfrac{3^{3/6}}{3^{1/6}} = 3^{2/6} = 3^{1/3} = \sqrt[3]{3}$

25) $\sqrt[3]{a^2\sqrt{a\sqrt[3]{a^2}}} = \sqrt[3]{a^2\sqrt{a\cdot a^{2/3}}} = \sqrt[3]{a^2\cdot a^{1/2}\cdot a^{2/6}}$

$= a^{2/3}\cdot a^{1/6}\cdot a^{2/18} = a^{2/3}\cdot a^{1/6}\cdot a^{1/9}$

$= a^{\frac{12+3+2}{18}} = a^{17/18} = \sqrt[18]{a^{17}}$

26) $\sqrt{a\sqrt{a\sqrt{a}}} = \sqrt{a\sqrt{a\cdot a^{1/2}}} = \sqrt{a\cdot a^{1/2}\cdot a^{1/4}} = a^{1/2}\cdot a^{1/4}\cdot a^{1/8}$

$= a^{\frac{4+2+1}{8}} = a^{7/8} = \sqrt[8]{a^7}$

27) $\dfrac{a}{\sqrt[3]{a}} = \dfrac{a^{3/3}}{a^{1/3}} = a^{2/3} = \sqrt[3]{a^2}$

29) $\log_2 8 = x$

$2^x = 8$

$\log 2^x = \log 8$

$x \cdot \log 2 = \log 8$

$x = \frac{\log 8}{\log 2}$

$x = 3$

$2 \cdot 2 \cdot 2 = 8$

$\sqrt[3]{8} = 2$

30) $\log_3 \frac{1}{9} = x$

$3^x = 1/9$

$x \log 3 = \log 1/9$

$x = \frac{\log 1/9}{\log 3}$

$x = -2$

$3^{-2} = 1/9$ ✓

28) $$\frac{(3\sqrt{2} + 2\sqrt{3})^2}{(3\sqrt{2} - 2\sqrt{3}) \cdot (3\sqrt{2} + 2\sqrt{3})} = \frac{18 + 2 \cdot 3\sqrt{2} \cdot 2\sqrt{3} + 12}{((3\sqrt{2})^2 - (2\sqrt{3})^2)}$$

$$\frac{30 + 12\sqrt{2}\sqrt{3}}{18 - 12} = \frac{30 + 12\sqrt{6}}{6} = \frac{6(5 + 2\sqrt{6})}{6}$$

$$= 5 + 2\sqrt{6}$$

$$\frac{(3\sqrt{2} + 2\sqrt{3})(3\sqrt{2} - 2\sqrt{3})}{(3\sqrt{2} - 2\sqrt{3})(3\sqrt{2} - 2\sqrt{3})} = \frac{6}{30 - 2 \cdot 3\sqrt{2} \cdot 2\sqrt{3}}$$

$$= \frac{6}{30 - 12\sqrt{6}} = \frac{\cancel{6}}{\cancel{6}(5 - 2\sqrt{6})} = \frac{1}{5 - 2\sqrt{6}}$$

31) $\log_2 1 = x$

$$2^x = 1$$

$$x = \frac{\log 1}{\log 2} = \frac{0}{\log 2} = 0$$

$$x = 0$$

$$2^0 = 1$$

32) $\ln 1 = x$

$$\log_e 1 = x$$

$$e^x = 1$$

$$x \log e = \log 1$$

$$x = \frac{\log 1}{\log e}$$

$$x = 0$$

33) $\log_2 0{,}5 = x$

$$2^x = 0{,}5$$

$$x = \frac{\log 0{,}5}{\log 2}$$

$$x = -1$$

$$2^{-1} = 0{,}5$$

34) $\log_a a = x \quad (a > 0)$

$$a^x = a$$

$$x = \frac{\log a}{\log a}$$

$$x = 1$$

$$a^1 = a$$

35) $\log_a b^3 - 2 \cdot \log_a b + \frac{1}{2} \log_a 2b^4 \quad , (a, b > 0)$

$$\log_a b^3 - \log_a b^2 + \log_a \sqrt{2} \cdot b^2 =$$

$$\frac{\log_a b^3}{\log_a b^2} + \log_a \sqrt{2} b^2 = \log_a\left(b \cdot (\sqrt{2} b^2)\right) = \log_a \sqrt{2} b^3$$

$$= \frac{1}{2} \cdot \log 2 b^6 \qquad = \log_a 2^{1/2} b^3$$

36) $\lg 10^1 = x$

$10^x = 10$

$x = \frac{\log 10}{\log 10}$

$x = 1$

$10^1 = 10$

37) $\ln e = x$

$\log_e e = x$

$e^x = e$

$x = \frac{\log e}{\log e}$

$x = 1$

$\underline{e^1 = e}$

38) $\ln \sqrt{e} = x$

$\log_e e^{1/2} = x$

$\frac{1}{2} \log_e e = x$

$\frac{1}{2} \cdot e^x = e$

$x = \frac{1}{2} \cdot \frac{\log e}{\log e}$

$x = \frac{1}{2}$

$\ln \sqrt{e} = \frac{1}{2}$

39) $\log_e e^{-3a+b} = x$

$(-3a+b) \cdot \log_e e = x$

$-3a+b = x$

40) $\log_a b^2 \cdot \log_b a^3 = x \; (a, b > 0)$

$6 \cdot \underbrace{\log_a b \cdot \log_b a}_{1} = x$

$x = 6$

Gleichungen $x^2+px+q=0 \Rightarrow x=-\frac{p}{2}\pm\sqrt{\left(\frac{p}{2}\right)^2-q}$

41) $x^2-x-2=0$

$x_{1,2}=+\frac{1}{2}\pm\sqrt{\left(\frac{1}{-2}\right)^2+2}$

$x_{1,2}=+\frac{1}{2}\pm 1{,}5$

$x_1=-1$

$x_2=+2$

42) $3x^2+10x+3=0$

$x^2+\frac{10}{3}x+1=0$

$x_{1,2}=-\frac{5}{3}\pm\sqrt{\left(\frac{5}{3}\right)^2-1}$

$x_{1,2}=-\frac{5}{3}\pm\sqrt{\frac{16}{9}}$

$x_{1,2}=-\frac{5}{3}\pm\frac{4}{3}$

$x_1=-\frac{1}{3}$

$x_2=-\frac{9}{3}=-3$

43) $4x^2-4x+1=0$

$x^2-x+\frac{1}{4}=0$

$x_{1,2}=+\frac{1}{2}\pm\sqrt{\left(\frac{1}{2}\right)^2-\frac{1}{4}}$

$x_1=x_2=\frac{1}{2}$

44) $2x^2+2x+1=0$

$x^2+x+\frac{1}{2}=0$

$x_{1,2}=-\frac{1}{2}\pm\sqrt{\frac{1}{4}-\frac{1}{2}}$

$x_{1,2}=-\frac{1}{2}\pm\sqrt{-\frac{1}{4}}$

$\longrightarrow$ kein Ergebnis, da negativ unter der Wurzel

45) $\left(x+\frac{3}{4}\right)\left(x-\frac{1}{4}\right)=\frac{5}{16}$

$x^2-\frac{1}{4}x+\frac{3}{4}x-\frac{3}{16}-\frac{5}{16}=0$

$x^2+\frac{1}{2}x-\frac{1}{2}=0$

$x_{1,2}=-\frac{1}{4}\pm\sqrt{\left(\frac{1}{4}\right)^2+\frac{1}{2}}$

$-\frac{1}{4}\pm\sqrt{\frac{9}{16}}$

$-\frac{1}{4}\pm\frac{3}{4}$

$x_1=\frac{1}{2}$

$x_2=-1$

46) $(8+x)(8-x)+(6-x)(10+x)=76$

$(8^2-x^2)\cancel{0}+60+6x-10x-x^2=76$

$48-2x^2-4x=0 \quad | :(-2)$

$x^2+2x\cancel{+24}=0$

$x_{1,2}=-1\pm\sqrt{(1)^2+\cancel{24}}$

$x_{1,2}=-1\pm 5$

$x_1=4$

$x_2=-6$

47) $24x^2 + 27 - 54x = 0 \quad /:24$

$x^2 - \frac{54}{24} + \frac{27}{24} = 0$

$x^2 - \frac{9}{4} + \frac{9}{8} = 0$

$x_{1,2} = \frac{9}{8} \pm \sqrt{\left(\frac{9}{8}\right)^2 - \frac{9}{8}}$

$= \frac{9}{8} \pm \sqrt{\frac{81}{64} - \frac{9}{8}}$

$= \frac{9}{8} \pm \frac{3}{8}$

$x_1 = \frac{3}{2}$

$x_2 = \frac{6}{8} = \frac{3}{4}$

48) $(2x-3)^2 - (x-5)^2 = 80$

$4x^2 + 9 - 2 \cdot 2x \cdot (-3) - x^2 - 25 - 2 \cdot x \cdot (-5)$

$= 80$

$3x^2 - 12x + 10x - 16 = 80$

$3x^2 - 2x - 96 = 0 \quad /:3$

$x^2 - \frac{2}{3}x - 32 = 0$

$x_{1,2} = \frac{1}{3} \pm \sqrt{\frac{1}{9} + 32}$

$= \frac{1}{3} \pm \sqrt{\frac{289}{9}}$

$= \frac{1}{3} \pm \frac{17}{3}$

$x_1 = 6 \qquad x_2 = -\frac{16}{3}$

49) $\frac{10x-1}{9} + \frac{6x-1}{5} - \frac{1}{x} - 2x + 1 = 0$

$\frac{5x(10x-1) + 9x(6x-1) - 45 - 90x^2 + 45x}{45x} = 0$

$\frac{50x^2 - 5x + 54x^2 - 9x - 45 - 90x^2 + 45x}{45x} = 0$

$\frac{14x^2 + 31x - 45}{45x} = 0 \quad / \cdot 45x$

49) $\dfrac{14x^2+31x-45}{45x}=0 \quad / \cdot 45x$

$$14x^2+31x-45=0$$

$$x^2+\frac{31x}{14}-\frac{45}{14}=0$$

$$x_{1,2}=-\frac{31}{28}\pm\sqrt{\left(\frac{31}{28}\right)^2+\frac{45}{14}}$$

$$x_{1,2}=-\frac{31}{28}\pm\sqrt{\frac{961}{784}+\frac{45}{14}}$$

$$\frac{3481}{784}$$

$$x_{1,2}=-\frac{31}{28}\pm\frac{59}{28}$$

$$x_1=1$$

$$x_2=-\frac{90}{28}=-\frac{45}{14}$$

50) $\dfrac{5+2x}{3-2x}-\dfrac{4-3x}{x}=\dfrac{2x}{x-1}$

$$\frac{(5x+2x^2)-(12-8x-9x+6x^2)}{(3-2x)\cdot x}=\frac{2x}{x-1}$$

$$\frac{5x+2x^2-12+8x+9x-6x^2}{3x-2x^2}=\frac{2x}{x-1}$$

$$\frac{-4x^2+22x-12}{3x-2x^2}-\frac{2x}{x-1}=0$$

$$\frac{(-4x^2+22x-12)(x-1)-(2x)(3x-2x^2)}{(3x-2x^2)(x-1)}=0$$

$$\frac{-4x^3+4x^2+22x^2-22x-12x+12-6x^2+4x^3}{3x^2-3x-2x^3+2x^2}=0$$

$\rightarrow$ 50) $\frac{20x^2 - 34x + 12}{-2x^3 + 5x^2 - 3x} = 0$

Erklärung:
der Nenner kann weggelassen werden, da nicht durch Null geteilt werden kann, deshalb muss der Zähler Null sein.

$20x^2 - 34x + 12 = 0$

$x^2 - \frac{17}{10}x + \frac{3}{5} = 0$

$x_{1,2} = \frac{17}{20} \pm \sqrt{\left(\frac{17}{20}\right)^2 - \frac{3}{5}}$

$x_{1,2} = \frac{17}{20} \pm \sqrt{\frac{49}{400}}$

$x_{1,2} = \frac{17}{20} \pm \frac{7}{20}$

$x_1 = \frac{6}{5}$

$x_2 = \frac{1}{2}$

$x^2 \cdot x^2$

51) $x^4 + 3x^2 - 4 = 0$ für $x^2 = x$

$x^2 + 3x - 4 = 0$

$x_{1,2} = -\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 + 4} = -\frac{3}{2}$

$= -\frac{3}{2} \pm \sqrt{\frac{25}{4}}$

$= -\frac{3}{2} \pm \frac{5}{2}$

$x_1 = 1$ $\quad \left[x_2 = -4\right] \rightarrow$ nicht möglich, da nicht 0

$1^4 + 3 \cdot 1^2 - 4 = 0 \rightarrow$ richtig

$(-1)^4 + 3 \cdot (-1)^2 - 4 = 0 \rightarrow$ richtig, da Potenzen gerade sind

52) $2^x = 16$

$x \log 2 = \log 16$

$x = \frac{\log 16}{\log 2}$

$x = 4$

$\rightarrow 2 \cdot 2 \cdot 2 \cdot 2 = 16$ ✓

53) $3^x = \frac{1}{3}$ $/\log$

$x \log 3 = \log \frac{1}{3}$

$x = \frac{\log \frac{1}{3}}{\log 3}$

$x = -1$

$3^{-1} = \frac{1}{3}$

54) $\left(\frac{1}{2}\right)^{x+1} = 8^{x-4}$ $/\log$

$(x+1)\log \frac{1}{2} = (x-4) \cdot \log 8$

$x+1 = \frac{(x-4)\log 8}{\log \frac{1}{2}}$

$\frac{x+1}{x-4} = \frac{\log 8}{\log \frac{1}{2}}$

$\frac{x+1}{x-4} = -3$

$x+1 = -3(x-4)$

$x+1 = -3x+12$

$4x = 11$

$x = \frac{11}{4}$

55) $e^{2x^2-1} = 1$

$2x^2 - 1 \log e = \log 1$

$2x^2 - 1 = \frac{\log 1}{\log e}$

$2x^2 - 1 = 0$

$2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \sqrt{\frac{1}{2}}$

$x = \pm\sqrt{\frac{1}{2}}$

56) $\left(\frac{3}{4}\right)^{3x+4} = \left(\frac{16}{9}\right)^{2x+1}$

$3x+4 \log 3/4 = 2x+1 \log \frac{16}{9}$

$\frac{3x+4}{2x+1} = \frac{\log 16/9}{\log 3/4}$

$3x+4 = -2\,(2x+1)$

$3x+4 = -4x-2$

$7x = -6$

$x = -\frac{6}{7}$

57) $2 \lg x = \lg 4$

$\lg_{10} x = \frac{\lg 4}{2}$

$x = \cancel{10}^{\left(\frac{\lg 4}{\cancel{2}}\right)}$

$x_1 = -2$

58) $\log x^3 = 6 \longrightarrow \log_{10} x^3 = 6$

$10^6 = x^3$

$\sqrt[3]{1000000} = x$

$x = 100$

59) $\lg x^5 = \lg x^3 + 6$

$5\lg x - 3\lg x = 6$

$2\lg x = 6$

$\lg x^2 = 6 \longrightarrow 10^6 = x^2$

$\sqrt{10^6} = x$

$x = 1000$

60) $\ln(2x+3) = \ln(x-1) + 1$ $\qquad x > 1 \qquad \log_e x = \ln x$

$\ln(2x+3) - \ln(x-1) = 1$

$\ln \frac{2x+3}{x-1} = 1$

$e^1 = \frac{2x+3}{x-1} \quad / \cdot (x-1)$

$ex - e = 2x + 3$

$ex - 2x = 3 + e$

$x(e-2) = 3 + e$

$x = \frac{3+e}{e-2}$

61) $\sqrt{x+2} - \sqrt{x-6} = 2 +$

$\sqrt{x+2} = (2 + \sqrt{x-6}) \quad / ()^2$

$x + 2 = 4 + 2 \cdot 2 \cdot \sqrt{x-6} + x - 6$

$4 = 4\sqrt{x-6}$

$\sqrt{x-6} = 1 \quad / ()^2$

$x = 7$

62) $\sqrt{1+x\cdot\sqrt{x^2+24}} = x+1 \quad /()^2$

$1+x\cdot\sqrt{x^2+24} = x^2+2x+1 \quad /:1+x$

$\sqrt{x^2+24} = \frac{x^2+2x+1}{1+x}$

$\sqrt{x^2+24} = \frac{(x+1)\cancel{(x+1)}}{\cancel{x+1}}$

$\sqrt{x^2+24} = x+1 \quad /()^2$

$x^2+24 = x^2+2x+1$

$-2x = -23$

$x = \frac{23}{2}$

$\sqrt{12{,}5\cdot\sqrt{12{,}5^2+24}}$

$\sqrt{12{,}5\cdot 12{,}5}$

$12{,}5 = 12{,}5$

63) $\sqrt{x+1} + \sqrt{2x+3} = 1$

$$\sqrt{2x+3} = 1 - \sqrt{x+1} \quad /()^2$$

$$2x+3 = 1-(2\cdot 1\cdot \sqrt{x+1}) + x+1$$

$$2x+3 = 1 - 2\cdot\sqrt{x+1} + x+1$$

$$x+1 = -2\cdot\sqrt{x+1}$$

$$-\frac{x+1}{2} = \sqrt{x+1} \quad /()^2$$

$$-\frac{(x^2+2x+1)}{4} = x+1$$

$$x^2+2x+1 = -4x-4$$

$$x^2+6x+5 = 0$$

$$x_{1,2} = -\frac{6}{2} \pm \sqrt{\frac{36}{4}-5}$$

$$= -\frac{6}{2} \pm \sqrt{4}$$

$$= -\frac{6}{2} \pm \frac{4}{2}$$

$x_1 = -1 \qquad x_2 = -\frac{10}{2} = -5$

Überprüfen:

$\sqrt{0} + \sqrt{1} = 1$

$1 = 1$ ✓

$\sqrt{-4} + \sqrt{-7} = 1$

↳ nicht möglich, da negativ unter der Wurzel

64) $\log_2(x+14)+\log_2(x+2) = 6$

$\boxed{\log_b x = z \text{ wenn } b^z = x}$

$\log_2(x^2+2x+14x+28) = 6$

$\log_2(x^2+16x+28) = 6$

$2^6 = x^2+16x+28$

$x^2+16x-36 = 0$

$x_{1,2} = -8 \pm \sqrt{64+36}$

$= -8 \pm \sqrt{100}$

$= -8 \pm 10$

$\underline{\underline{x_1 = 2}}$ $\boxed{x_2 = -18}$

→ nicht möglich da log(negativ) nicht erlaubt ???

Überprüfen:

$\log_2(16 \cdot 4) = 6$

$\log_2 64 = 6$

$2^6 = 64$ ✓

$\log_2(-4 \cdot -16)$

$\log_2 64 = 6$

$2^6 = 64$ ✓

65) $\log_4 \log_3 \log_2 x = 0$

$\log_3 \log_2 x = 4^0$

$\log_2 x = 3^1$

$x = 2^3 = 8$

66) $\dfrac{\lg(35-x^3)}{\lg(5-x)} = 3 \qquad | \cdot \lg(5-x)$

$\lg(35-x^3) = 3 \cdot \lg(5-x)$

$\lg(35-x^3) = \lg - 3 \cdot 5^2 \cdot x + 3 \cdot 5 \cdot x^2 - x^3$

$\lg(35-x^3) = \lg(-x^3+15x^2-75x+125)$

$\lg\left(\dfrac{35-x^3}{-x^3+15x^2-75x+125}\right) = 0$

$10^0 = \dfrac{35-x^3}{-x^3+15x^2-75x+125}$

$\cancel{-x^3}+15x^2-75x+125 = 35\cancel{-x^3}$

$15x^2-75x+90 = 0$

$x^2-5x+6 = 0$

$x_{1,2} = \dfrac{5}{2} \pm \sqrt{\dfrac{25}{4}-6}$

$= \dfrac{5}{2} \pm \sqrt{\dfrac{1}{4}}$

$= \dfrac{5}{2} \pm \dfrac{1}{2}$

$\underline{\underline{x_1 = 3}} \qquad \underline{\underline{x_2 = 2}}$

Prüfen: $\dfrac{\lg 8}{\lg 2} = 3$

$3 = 3$ ✓

$\dfrac{\lg 27}{\lg 3} = 3$

$3 = 3$ ✓